005383

NASA TECHNICAL NOTE

NASA TN D-3489

A DESIGN PROCEDURE FOR THE WEIGHT OPTIMIZATION OF STRAIGHT FINNED RADIATORS

by Dale W. Harris
Goddard Space Flight Center

and

R. J. Burian and J. J. Ketchman
Battelle Memorial Institute

20020328 105

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1966

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Goddard Space Flight Center
Greenbelt, Md.

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R. J. Burian and J. J. Ketchman

Battelle Memorial Institute
Columbus, Ohio

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ABSTRACT

Radiators for rejecting waste heat from power generators in space can be an important weight contributor to the total weight of space power systems. For the rejection of from a few hundred watts up to perhaps a few kilowatts of waste heat straight fin radiators are the most practical. In a recent study program of weight optimization of thermo-electric power generators, a technique was established which permits the rapid determination of the geometry of a minimum weight finned radiator system. From data presented in the literature, three design equations were derived which relate twelve geometric, thermal, environmental and material parameters of an idealized fin system with no base cylinder interaction. A fourth equation was derived to take into account the base cylinder interaction and to reduce the idealized design to the realistic case. Three families of curves and auxiliary tables were prepared to assist in the rapid reduction of the idealized design equations.

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INTRODUCTION

Probably the most critical yardstick used for measuring the suitability of any space vehicle or component is its weight. Because of the great expense of placing each pound of material in orbit, the weight of a space package is trimmed wherever possible without sacrificing performance or strength. Thus, the design of space power systems is directed toward developing a minimum weight device for the required power output. For systems developing large quantities of electrical power, in the several 100-watt and kilowatt range, the radiators usually take the form of large panels containing a network of passages through which the power converter coolant flows. In the case of smaller systems, the radiator usually takes the form of straight fins extending radially from a central body which contains the power conversion device. The waste heat is transmitted to the finned radiator from the conversion device by conduction. Several excellent papers which present analyses of the latter type of radiator have appeared in the literature. These analyses, however, do not lend themselves to rapid design of finned radiators. It is the purpose of this paper to present a technique based on these analyses whereby a weight optimized space radiator consisting of a finned, right circular cylinder can be readily evaluated.

ANALYSIS OF FINNED RADIATORS

Two noteworthy papers have been authored by Eckert, Irvine and Sparrow, References 1 and 2. In the first, analytical formulas were established for heat rejection by radiation from mutually irradiating straight fins intersecting at their base. Subsequently, in Reference 2, a group of curves were presented which lend themselves to use for design of straight finned radiators. These curves indicated the relationship between fin effectiveness (defined as the ratio of the actual heat rejected to the maximum ideally possible) and a conduction parameter which was a function of the fin conductivity, fin dimensions and temperature at the fin base. These curves were presented for various

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opening angles between adjacent fins and various values of surface emissivity. All analyses were performed for fins of rectangular cross section, i.e., the fin thickness was constant from the base to the tip. These analyses also assumed the case where incident heat from space was zero. It was then shown by Sparrow et al., that for a given emissivity and number of equally spaced fins there exists a minimum weight optimized value of the conduction parameter and, therefore, a specific value of fin effectiveness; however, the choice of the number of fins for weight optimization was not indicated.

Similar work was performed by Heaslet and Lomax Reference 3, and Karkelar and Chao, Reference 4. In their analyses, however, Karlekar and Chao introduced another dimensionless parameter in addition to those previously presented by Sparrow, et al. This term was a heat dissipation parameter which is a function of the heat actually rejected, the fin base temperature, the conductivity, and the profile area of the fins. The analysis considers not only fins of rectangular cross section, but fins of any trapezoidal profile from a rectangle to a triangle. In addition, the analysis considers the effect of incident radiation from space.

Karlekar and Chao also performed a weight optimized analysis of the parameters. They illustrated that there is an optimum number of fins for minimum radiator weight for a specific trapezoidal fin profile and values of emissivity and incident radiation from space. These data are summarized in Table 1. Also included in the table are optimized values of a fin parameter λ and a heat dissipation parameter ζ. These quantities are defined by Equations 7 and 8 in the next section. Optimum values of N, λ, and ζ are arrived at by solving a set of simultaneous, nonlinear algebraic equations which result from a finite difference formulation of the temperature field in the fins. It was also shown by Karlekar and Chao that fins of triangular profile will reject more heat per unit weight than will a fin of any other trapezoidal profile. Comparison of the data presented by Reference 4 with that presented by References 1 and 2 revealed that the two works are entirely compatible.

Table 1

Optimum Number of Fins.*

Trapezoidal Shape Factor τ	Emissivity ϵ	Incident Radiation β	Optimum Fin Number N_{opt}	Optimum Fin Parameters	
				λ_{opt}	ζ_{opt}
0.99	1.00	0.00	4	0.95	1.8310
	0.90	0.00	5	1.15	1.7563
	0.90	0.25	5	1.13	1.7525
	0.90	0.50	4	1.00	1.6930
	0.75	0.00	6	1.31	1.6390
	0.50	0.00	7	2.00	1.4350
0.75	0.90	0.00	5	1.17	1.6980
0.00	1.00	0.00	5	1.15	1.6245
	0.90	0.00	5	1.28	1.5620
	0.90	0.25	5	1.27	1.5585
	0.90	0.50	5	1.25	1.5040
	0.75	0.00	6	1.58	1.4690
	0.50	0.00	8	2.35	1.2850

*From Karlekar and Chao (Reference 4).

Although they can be used to design straight finned radiators, the results of the analyses discussed above, because of the manner of presentation, do not lend themselves to rapid calculation of the factors of prime interest to the design engineer, namely, what will be the size and weight per unit heat rejected for the fin system. Therefore, building on these analyses, a study was performed to establish readily usable engineering design equations.

DEVELOPMENT OF DESIGN EQUATIONS

In the development of the design equations it was necessary to consider all important parameters which might effect the heat rejection ability and weight of the radiator. These included thermal, physical, and structure properties of the radiator material. The parameters considered (not necessarily all independent) included the following:

q total heat rejected by the fin system

W total weight of the fin systems

L fin height (radially)

b fin width

t fin thickness at the base

T_b absolute temperature of the fin at the base

N number of fins in the system (assumed to be evenly spaced in the 360 degree angle around a heat source of negligible radial dimension compared to the fin height)

k thermal conductivity of the fin material

ρ density of the fin material

ϵ surface emissivity

τ fin profile parameter defined by Karlekar and Chao as $\tau = 1 - (t_m/t)$ where t_m is the thickness of the fin at the tips

θ parameter for incident radiation from space defined by Karlekar and Chao as $\theta = T_s/T_b$ where T_s is the effective absolute temperature of space, and any of several material strength parameters such as yield strength.

It will be noted that parameters which define the central body of space power generator are not included. Since the analyses were for a system of fins intersecting at their base (i.e., no central body) with a line heat source at this intersection, the initial development of the design equations were for this idealized geometry. Subsequently, an equation was developed which modifies these idealistic equations to the practical case.

The parameter, θ, may be expressed by the equation

$$\theta = \left(\frac{H}{\sigma T_b^4} \right)^{1/4} ,$$

corresponds to a temperature of about 425°K (720°R). As pointed out by Karlekar and Chao, θ of 0.25 or less will not effect the numerical results of the analyses while values of θ greater than 0.50 have significant influence.

In order to more easily permit the parametric analysis in terms of prime interest to the design engineer, it was felt desirable to reduce the number of variables. It has been noted that in the works of Karlekar and Chao a fin of triangular profile will reject more heat per unit weight than will a trapezoidal fin of any other profile. Thus, for a minimum weight design, a triangular fin profile should be used, and the parameter, τ, should be a constant. In addition, the amount of incident radiation is dictated by the mission and not by the design of the radiator, thus θ can be defined as a constant. The emissivity, ϵ, should be as high as possible for maximum heat rejection. It can be taken as a constant parameter by assuming that a suitable surface coating can be applied to the radiator if the material itself does not possess a suitable emissivity. Having been able to define τ, θ, and ϵ, one is able (from Table 1) to select the number of fins, N, required to result in a minimum weight idealized finned radiator. Consequently, four terms can quite readily be removed from the above list for parametric evaluation. In addition, three other parameters, all material property terms, can be eliminated as variables by development of a material selection criteria.

In order for a material to be a strong candidate for use as a low weight radiator, it should have a high thermal conductivity, k, a low density, ρ, and possess adequate strength at the radiator operation temperature. If the strength criteria is momentarily ignored, the worth of any material for use as a radiator can be shown by a materials parameter, the ratio of its conductivity to its density, k/ρ. Thus, for an analysis of a minimum weight radiator, a material with the greatest materials parameter should be used effectively removing k and ρ from the list of variables. It is possible that such an arbitrary selection could result in use of a material which would not possess the necessary strength for the radiator designed by the parametric analysis. Therefore, selection should be made while considering the strength limitations of the material. The most satisfactory way to select a material is to estimate what stresses the final radiator will experience, choose a material, and verify this choice after the radiator size and weight are determined.

The number of variables for consideration were thus reduced from 13 to 6. Those remaining are T_b, W, q, L, b, and t. These were combined into three design equations with three constants, μ, ϕ, and ψ, which contain the other parameters considered except a strength parameter. The three design equations are:

$$\frac{q}{w} = \left(\frac{q}{b} \mu\right)^{-2} T_b^9 \times 10^{-22} \, , \tag{1}$$

$$L = \left(\frac{q}{b} \phi\right) T_b^{-4} \times 10^{10} \, , \tag{2}$$

and

$$t = \left(\frac{q}{b} \psi\right)^2 T_b^{-5} \times 10^{10} \, , \tag{3}$$

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$$L = \left(\frac{q}{b}\,\phi\right) T_b^{-4} \times 10^{10} , \tag{2}$$

and

$$t = \left(\frac{q}{b}\,\psi\right)^2 T_b^{-5} \times 10^{10} , \tag{3}$$

where

$$\mu = \left(\frac{\rho}{k\,\sigma^2\,\zeta^3}\right)^{1/2} 10^{-11} , \tag{4}$$

$$\phi = \left[\frac{\lambda}{N(2-\tau)}\right]^{1/3} \left(\frac{1}{\zeta\,\sigma}\right) 10^{-10} , \tag{5}$$

$$\psi = \left[\frac{2}{\sigma\,k\,N^{2/3}\,(2-\tau)^{2/3}\,\lambda^{1/3}\,\zeta^2}\right]^{1/2} 10^{-5} , \tag{6}$$

$$\lambda = \frac{2T_b^3\,L^2}{kt} , \tag{7}$$

and

$$\zeta = \frac{q}{b}\left[\sigma^2\,T_b^9\,k\,NL\,\frac{t}{2}\,(2-\tau)\right]^{-1/3} . \tag{8}$$

This is a very desirable group of relationships since in the design of a radiator, the usual input data includes q, T_b, and b. Curves of these functions have been plotted in Figures 1, 2, and 3. To aid in the computation, values of μ, ϕ, and ψ were determined by computer for various materials, combinations of fin number, N, and emissivity, ϵ, and for the case where $\tau = 0.99$ (a near triangle), $\theta = 0$, and for average values of k over the usable temperature range of each material. Local optimum values of λ and ζ, given by Karlekar and Chao, were used in the computations for values of N other than N_{opt}. The over-all optimum parameters corresponding to N_{opt} are those given in Table 1. If the local optimum values of λ and ζ are desired, Equation 4 together with Equation 5 or 6 may be used to back-calculate these parameters.

Nine different materials were selected for potential use at various temperatures over the range from room temperature to 1140°K (2050°R). These alloys were selected based on their materials parameter and their yield strength at temperature. They included the metals and/or alloys of aluminum, beryllium, copper, magnesium and molybdenum. These tables can also be used for the case where incident radiation from space is other than zero (i.e., $\theta \neq 0$). In this case, the heat absorbed is assumed and added to the heat to be rejected, q. After the design is performed, the accuracy of the assumption can be verified and adjusted if necessary. For any fin configuration, i.e., number of fins, this technique will result in a slightly conservative (oversize) design since the external heat which is absorbed is immediately ready for reirradiation from the fin surface, whereas in the analysis the heat enters the fin at the base and must be conducted a finite distance along the fin before it can be radiated.
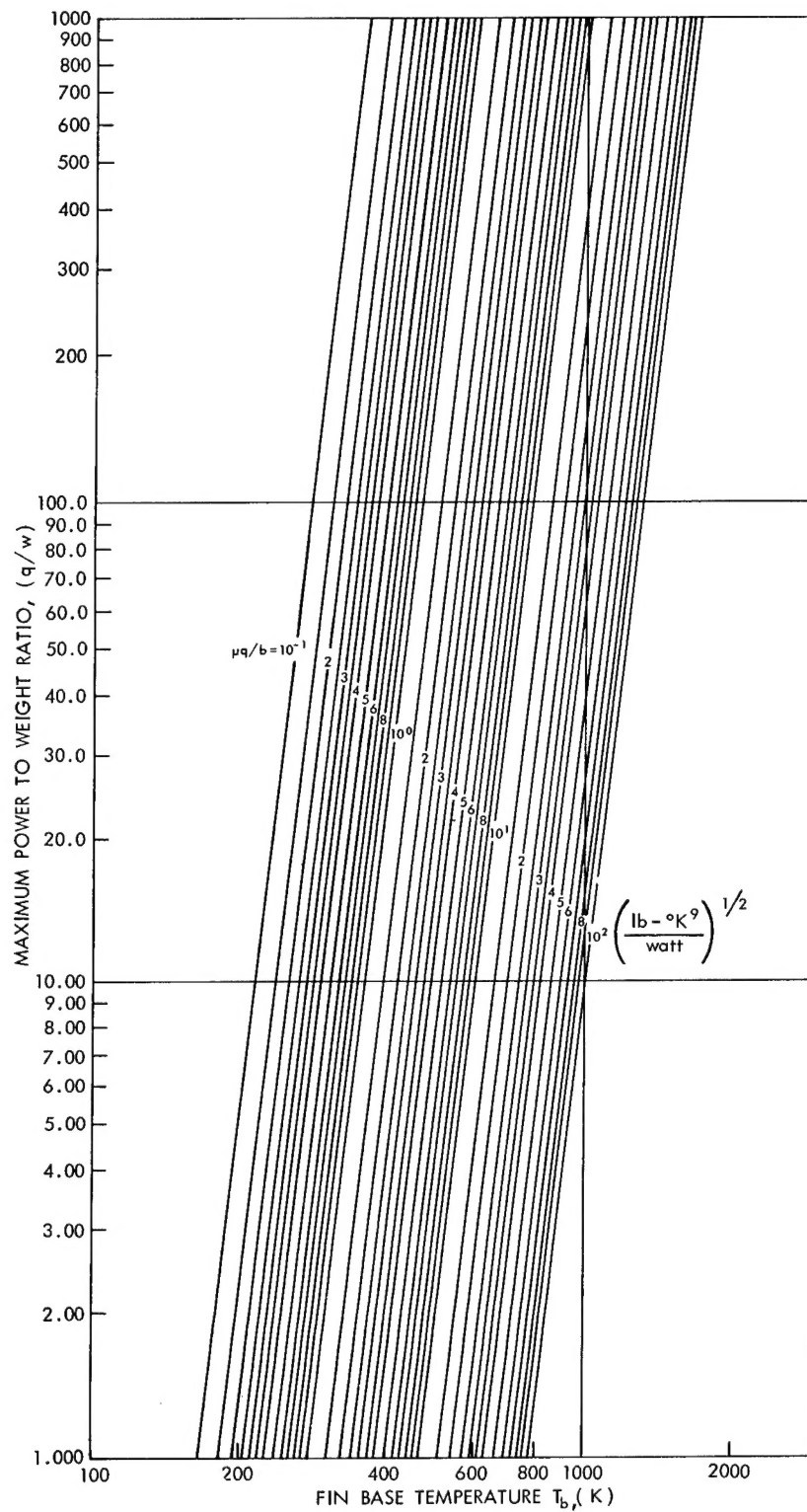
Figure 1—Maximum power to weight ratio of the fin system as a function of fin base temperature for values of $\mu\, q/b$ from $1/10$ to $10^2 \left[(lb - °K^9)/w\right]^{1/2}$.

Figure 2—Optimum fin height as a function of heat rejection per unit width (q/b), fin length parameter (ϕ) versus fin base temperature.

FIN THICKNESS t_r (cm)

10
8.0
6.0
4.0

2.0

1.0
0.8
0.6
0.4

0.2

$\psi q/b = 10^0$

0.1
.08
.06
.04

.02

.01
100   200   400   600 800 1000   2000   4000   6000 8000 10,000

FIN BASE TEMPERATURE T_b, ($^\circ$K)

2
3
4
5
6
8
10^1
2
3
4
5
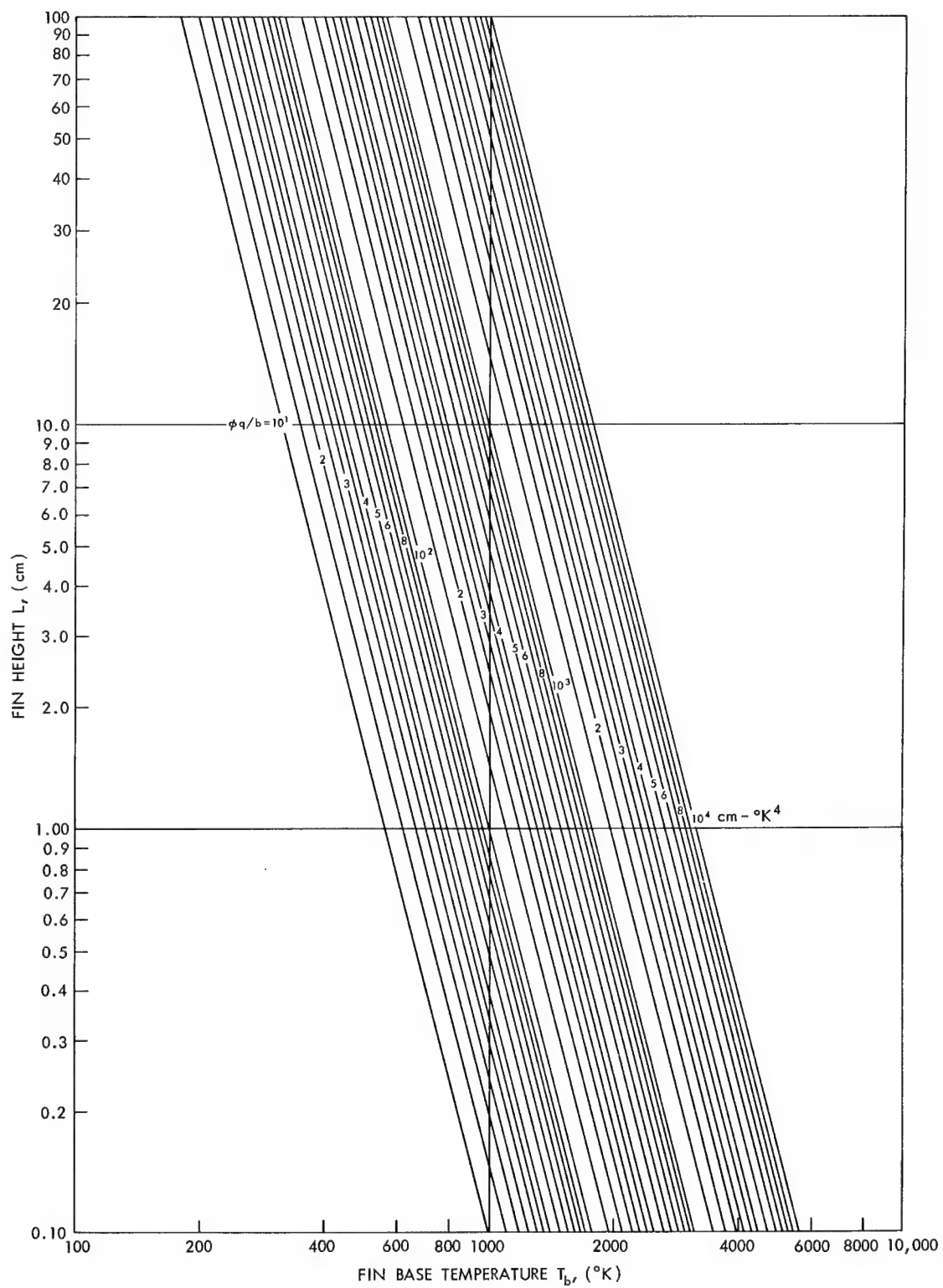6
8
10^2
2
3
4
5
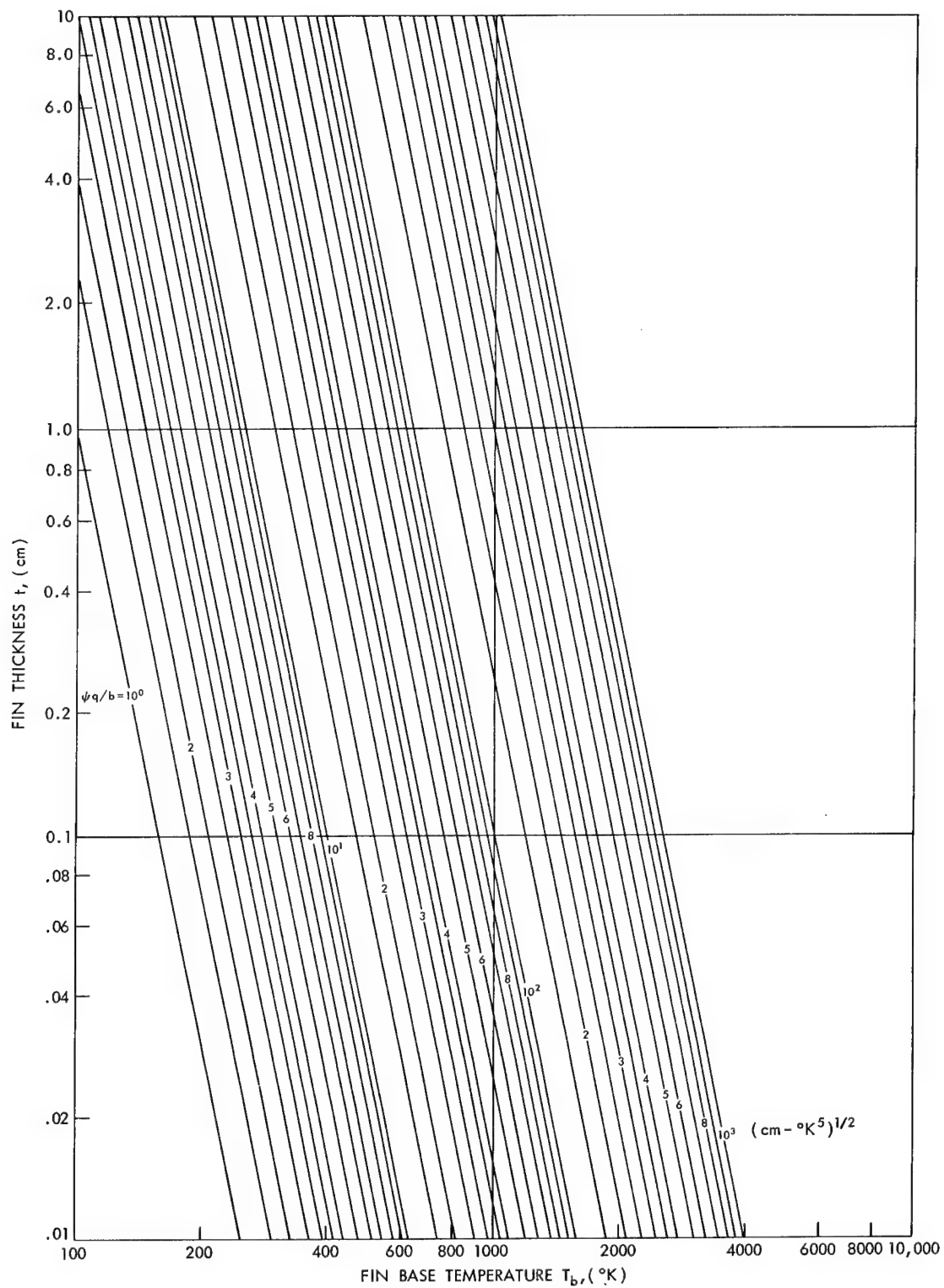6
8
10^3 $(cm - ^\circ K^5)^{1/2}$

Figure 3—Optimum fin thickness as a function of heat rejection per unit width (q/b), fin
thickness parameter (ψ) versus fin base temperature.

The results of this study delineated an important geometric relationship which is recognized by space vehicle designers but was not discussed by the analysts mentioned nor readily apparent in their results. Although Sparrow, et al., and Karlekar and Chao indicated the weight optimization possibilities of the straight-finned system, and in addition, Karlekar and Chao indicated the optimization of the number of fins, neither analyst discussed the interdependence of the fin dimensions. This interdependence is apparent from the three design equations (1, 2, and 3) where it is seen that the design of an idealized, minimum-weight optimized fin system does not permit the arbitrary selection of more than one of the three dimensions of the fin, L , b or t . The selection of any one of these dimensions along with specific fin-base temperature and the amount of heat to be rejected immediately specifies the other two dimensions.

USE OF THE DESIGN CURVES FOR AN IDEALIZED CASE

In order to illustrate the many possible uses of the design curves, first consider two examples of a radiator design. In the first case, a lowest possible weight radiator is to be designed thus requiring that an optimized fin configuration be used. In the second case, the best design for an off-optimum fin configuration is determined for comparison. These two cases were chosen to emphasize the fact that this analysis can be used to design for other than the optimum number of fins. In these cases the resulting weight will be the minimum attainable for the chosen fin configuration, but not necessarily the lowest possible weight. As stated, the optimum number of fins must be used to obtain the lowest possible radiator weight. Assume that for both examples the radiation from space is negligible (θ = 0), a near triangular fin profile will be used (τ = 0.99) and a surface coating can be applied to the radiator which will maintain an emissivity of about 0.9. The coating is assumed sufficiently thin so that temperature drop through the coating is negligible. Also assume for both cases that the radiator must reject 300 watts of heat at a fin base temperature of 127°C (400°K) and from design considerations of the power conversion device, the fin can be a maximum of 20 cm wide. Then, for an operating temperature of 127°C, the magnesium-thorium alloy, HM21A-T8, is suitable for use from the strength consideration. From Table 1 it is seen that for θ = 0, τ = 0.99, and ϵ = 0.9, the optimum number of fins is 5. From Table A-4, for θ = 0, τ = 0.99, ϵ = 0.9, and N = 5, it is seen that

$$\mu = 0.040478 ,$$

$$\phi = 6.1526 ,$$

and

$$\psi = 1.6516 .$$

Then

$$\frac{q}{b} \mu = \left(\frac{300}{20}\right) (0.040478) = 0.607 ,$$

$$\frac{q}{b} \phi = \left(\frac{300}{20}\right) (6.1526) = 92.3 ,$$

$$\frac{q}{b} \psi = \left(\frac{300}{20}\right) (1.6516) = 24.8 .$$

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From Figures 1, 2, and 3 for T_b = 400°K, it is seen that the optimum triangular profile fin will exist when

$$\frac{q}{W} = 71 ,$$

$$L = 36.5 ,$$

and

$$t = 0.59 .$$

Then for q = 300 watts, the optimized weight is 4.22 lb.

In the second, off-optimized case, assume that from undefined considerations it is more desirable to design a generator with eight fins instead of five. All other conditions are the same as above. Then,

$$\frac{q}{b} = \frac{300}{20} = 15 ,$$

$$\mu = 0.043228 ,$$

$$\phi = 5.8263 ,$$

$$\psi = 1.4330 ,$$

$$\frac{q}{W} = 62 ,$$

$$W = 4.84 ,$$

$$L = 33.5 ,$$

and

$$t = 0.42 .$$

This then is the lowest weight radiator that can be designed to reject 300 watts of heat using eight fins. It is seen that in the selection of eight fins in preference to five, a weight penalty of 0.62 lb or 14.7 percent of the optimum weight is incurred. It should be noted that at higher heat-rejection

levels with larger fin systems, the penalty in weight for deviating from the optimum configuration may be very severe.

The curves presented also show quite vividly the dependence of the fin length, thickness and weight on the fin width. This can be illustrated by another example. Consider a variation of the second case above. Assume that with the change to the off-optimum number of eight fins, the fin base can be lengthened to 25 cm. Then q/b = 12 w/cm and for

$$T_b \ = \ 400 \ ,$$

$$\frac{q}{W} \ = \ 98 \ ,$$

$$W \ = \ 3.06 \ ,$$

$$L \ = \ 27.5 \ ,$$

and

$$t \ = \ 0.28 \ ,$$

which represents a weight savings of 1.16 lb or 27.5 percent from the optimized configuration with five 20-cm-wide fins.

On the other hand, if b must remain 20 cm for the off-optimum fin configuration using eight fins, the same weight savings can be achieved from the five-fin case by raising the rejection temperature to about 415°K (142°C) as determined from Figure 1 for q/w = 98 w/lb, q/b = 15 w/cm, and $q/b \ \mu$ = 0.607 (lb°K$\sqrt[9]{}$w)$^{1/2}$. A great variety of other operations can be rapidly performed with these curves by varying the input of the factors considered in the analysis.

These results also show the dependence of the length, thickness, and weight of an idealized fin system on the heat rejection temperature. The ratio of the total heat rejected to the fin width, q/b, is quite significant indicating that fin widths should be large for low fin system weight. One other fact revealed is that the fin height (L) for minimum-weight finned radiator is independent of the fin material considered. This is apparent when it is seen that the expression for the fin-height parameter, ϕ, is independent of k and ρ. Thus, the choice of material only affects the fin-base thickness and the specific power, q/w of the radiating straight-fin system.

MODIFICATION FOR PRACTICAL RADIATOR DESIGN

The above analyses were performed for an idealized radiator fin system in which no thermal interaction with a fin base surface occurs. In a realistic generator design, however, the fins are

attached to the surface of the generator case which has a finite size and area for interaction with the fins. If the effect of this base surface were ignored and the radiator fins designed according to the idealistic equations, the fins would be oversized since no credit would be taken for heat radiated from the base surface. In order to modify the idealistically designed fins to the real case an equation was derived from considerations of the temperature profile of ideal fins.
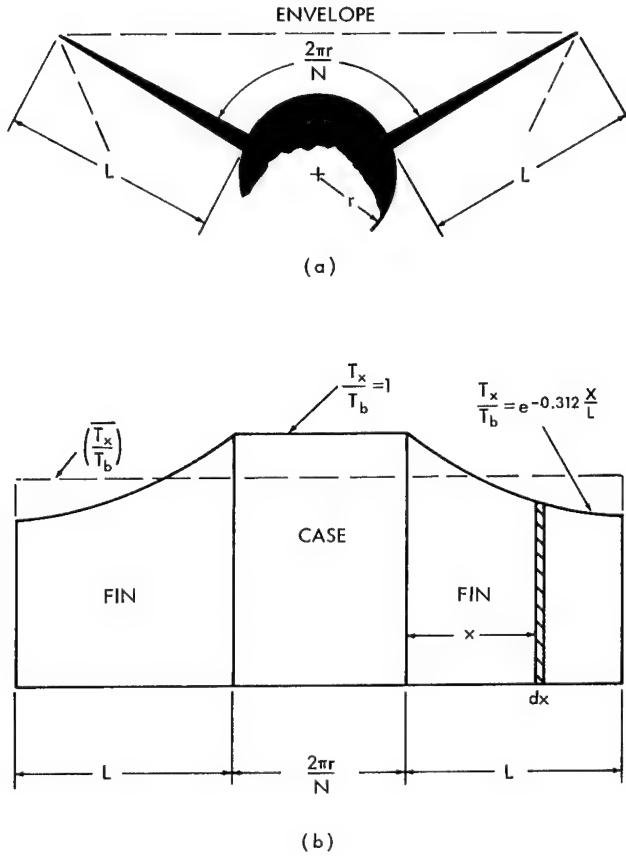

(a)

(b)

Figure 4—Fin and case temperature profiles.

Temperature profiles for ideal optimized fin systems are presented by Reference 4. This presents a dimensionless temperature ratio T_x/T_b as a function of position on the fin, x/L, where T_x is the temperature at any point x measured outward from the fin base. Three curves in Reference 4 are presented for fins of triangular shape factor ($\tau = 0.99$), and for $\theta = 0$. These curves are nearly superimposed and may be closely approximated by one curve. The equation for this curve was found to be

$$\frac{T_x}{T_b} = e^{-0.312 \, x/L} . \tag{9}$$

If it is assumed that this same temperature profile exists when the fins are put on a base surface of finite size, then a temperature diagram may be drawn for a generator section between any two fins, Figure 4. The temperature profile shown assumes the entire case to be radiating at the fin base temperature. Then an average temperature ratio, T_x/T_b may be established by letting T_x/T_b equal unity at the fin base and along the base surface.

$$\frac{\overline{T}_x}{T_b} = \frac{\int \frac{T_x}{T_b} \, dx}{\int dx} = \frac{2 \int_{[Fin]} \frac{T_x}{T_{b\,Fin}} \, dx + \int_{[Case]} \frac{T_x}{T_{b\,Case}} \, dx}{2 \int_{[Fin]} dx + \int_{[Case]} dx} ,$$

$$\int_{[Fin]} \frac{T_x}{T_{b\,Fin}} \, dx = \int_0^L e^{-0.312 \, x/L} \, dx ,$$

$$\int_{[Case]} \frac{T_x}{T_{b\,Fin}} \, dx = \int_0^{2\pi r/N} dx ,$$

$$\int_{[\text{Fin}]} dx = \int_0^L dx \, ,$$

and

$$\int_{[\text{Case}]} dx = \int_0^{2\pi r/N} dx \, .$$

Substituting and integrating gives

$$\frac{\bar{T}_x}{T_b} = \frac{0.8585\, L + \dfrac{\pi r}{N}}{L + \dfrac{\pi r}{N}} \, .$$

$\bar{T}_x$ can now be regarded as an effective envelope temperature, T_e, at which the envelope enclosing the fin tips radiates heat. Thus

$$T_e = T_b \left(\frac{0.8585\, L + \dfrac{\pi r}{N}}{L + \dfrac{\pi r}{N}} \right) . \tag{10}$$

Referring to Figure 4 the envelope area can be expressed as

$$A_e = 2Nb\,(L + r)\sin\left(\frac{360}{2N}\right) . \tag{11}$$

Then the heat radiated to space at absolute zero temperature from a realistic radiator system is

$$Q_{\text{TOT}} = 2Nb\,(L + r)\sin\left(\frac{360}{2N}\right)\sigma \,\epsilon\, T_b^4 \left(\frac{0.8585\, L + \dfrac{\pi r}{N}}{L + \dfrac{\pi r}{N}} \right)^4 . \tag{12}$$

In most design cases, Q_{TOT}, the total heat that must be rejected, is known. Then the proper fin length, L, may be determined by iteration of Equation 12. When L is established, Equation 2 or the curves in Figure 2 can be used to determine q/b which more precisely represents q_{Fin}/b. Using this value of q/b the weight of the fins and the fin base thickness may subsequently be determined from Equations 1 and 3 or Figures 1 and 3.

Equation 12, which modifies the ideal design equations to a real radiator situation, constitutes only an approximation since it is based on the temperature profile of an ideal fin. It can be shown that the equation meets within a few percent the boundary condition of reducing to the ideal equation as the area of the base surface approaches zero although its accuracy for other cases cannot be

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verified. Since the fin width, b, must remain finite, the area of the base surface would approach zero as the radius, r, of the fin-base cylinder approaches zero. When r = 0, Equation 12 reduces to

$$\frac{Q_{TOT}}{b} = 2NL\, \sigma\, \epsilon\, T_b^4 \sin\left(\frac{180}{N}\right)(0.8585)^5 \,.$$ (13)

Table 2

Percent Variation versus Number of Fins.

Number of Fins	$\dfrac{\frac{Q_{TOT}}{b} - \frac{q}{b}}{\frac{q}{b}}$ (percent)
3	+1.52
4	+1.02
5	+0.25
6	-1.72
8	-1.12

Equations 13 and 2 were solved for Q_{TOT}/b and q/b, respectively, for the case of ϵ = 0.9. Comparing the results reveals that for each radiator system between 3 and 8 fins, the solutions vary by less than 2 percent as shown in Table 2. These data indicate that at the limit of r = 0 for less than the optimum number of 5 fins, the radiator system is slightly conservative in design and for greater than 5 fins the system is slightly optimistic.

Equation 12 defines the heat rejected from an envelope described by planes intersecting at the fin tips. This equation, however, is no longer valid when the length of the fins relative to the radius of the base cylinder is sufficiently small so that the envelope intersects the base cylinder. Thus, Equation 12 can be used only for the range

$$0 < r < \frac{L}{\sec\left(\frac{180}{N}\right) - 1} \,.$$

This same technique could also be followed to establish a modification to the ideal design for cases of θ other than zero and τ other than 0.99.

CONCLUDING REMARKS

As expected, the curves presented show quite vividly the dependence of the fin length, thickness, and weight on the rejection temperature. However, the ratio of the heat rejected to the fin width (q/b) is just as significant. As illustrated in the example, the fin weight is heavily influenced by changes in fin width. Therefore, in the design of radiating systems, one should seek to make the fin width as large as possible if the geometry so permits. It can also be seen that if either the fin length or thickness must remain fixed, a compromise can be made with the fin width to permit operation at a given temperature. Inspection of the expression for ϕ reveals that this parameter is not dependent upon materials properties. Rather it is a function only of thermal and geometric parameters. The results presented then put forth a useful tool with which the design engineer with

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sufficient familiarity with the curves can rapidly evaluate the various playoffs encountered in radiator design.

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Appendix A

Optimized Fin Parameters of Various Materials
(Battelle Memorial Institute)

Table A1

Optimized Fin Parameters for Magnesium.

Thermal Conductivity, $k = 1.54$ w/(cm) (°K)
Density, $\rho = 0.0038$ lb/cm^3
$k/\rho = 405.0$ w-cm^2/(lb) (°K)
$\tau = 0.99$
$\theta = 0$
Maximum Usable Temperature $= 535$°K for a Yield Stress of 3400 psi

Emissivity, ϵ	Number of Fins, N	Fin–Length Parameter, ϕ, cm^2–°K^4/w	Fin–Thickness Parameter, ψ, (cm^3–°K^5/w^2)$^{1/2}$	Fin–Weight Parameter, μ, (cm^2–lb–°K^9/w^3)$^{1/2}$
1.00	2	8.2855	2.5974	0.046361
	3	6.6536	1.8833	0.036893
	4	5.9651	1.6615	0.035586
	5	5.7623	1.5195	0.035762
	6	5.6200	1.4417	0.036708
	8	5.5936	1.3268	0.038916
0.90	2	9.2663	2.7287	0.051506
	3	7.0492	1.9953	0.040232
	4	6.4434	1.7154	0.038184
	5	6.1526	1.5576	0.037881
	6	5.9094	1.4707	0.038398
	8	5.8263	1.3514	0.040454
0.75	2	11.0350	2.9960	0.061714
	3	8.8649	2.0869	0.047189
	4	7.2735	1.8335	0.043361
	5	6.7303	1.6543	0.042077
	6	6.4795	1.5370	0.042019
	7	6.2896	1.4536	0.042290
	8	6.2419	1.3930	0.043160
0.50	2	17.4190	3.7864	0.097992
	3	12.0580	2.5562	0.067409
	4	10.2320	2.1060	0.059075
	6	8.5048	1.6885	0.052886
	7	8.0949	1.5540	0.051291
	8	7.9412	1.4949	0.052243
	9	7.7944	1.4332	0.052633
	10	7.9902	1.4430	0.056556

Table A2

Optimized Fin Parameters for Beryllium.

Thermal Conductivity, k = 1.50 w/(cm) (°K)
Density, ρ = 0.0041 lb/cm^3
k/ρ = 366.0 w-cm^2/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 810°K for a Yield Stress of 4000 psi

Emissivity, ϵ	Number of Fins, N	Fin-Length Parameter, ϕ, cm^2-°K^4/w	Fin-Thickness Parameter, ψ, (cm^3-°K^5/w^2)$^{1/2}$	Fin-Weight Parameter, μ, (cm^2-lb-°K^9/w^3)$^{1/2}$
1.00	2	8.2855	2.6279	0.048369
	3	6.6536	1.9054	0.038491
	4	5.9651	1.6810	0.037128
	5	5.7623	1.5373	0.037311
	6	5.6200	1.4586	0.038298
	8	5.5936	1.3423	0.040602
0.90	2	9.2663	2.7606	0.053737
	3	7.0492	2.0186	0.041974
	4	6.4434	1.7354	0.039838
	5	6.1526	1.5759	0.039522
	6	5.9094	1.4879	0.040061
	8	5.8263	1.3672	0.042207
0.75	2	11.0350	3.0311	0.064387
	3	8.8649	2.1114	0.049233
	4	7.2735	1.8549	0.045240
	5	6.7303	1.6737	0.043900
	6	6.4795	1.5550	0.043839
	7	6.2896	1.4706	0.044122
	8	6.2419	1.4093	0.045029
0.50	2	17.4190	3.8307	0.102240
	3	12.0580	2.5861	0.070329
	4	10.2320	2.1306	0.061633
	6	8.5048	1.7083	0.055176
	7	8.0949	1.5722	0.053512
	8	7.9412	1.5124	0.054506
	9	7.7944	1.4500	0.054913
	10	7.9902	1.4599	0.059005

Table A3

Optimized Fin Parameters for Magnesium M1A.

Thermal Conductivity, k = 1.38 w/(cm) (°K)
Density, ρ = 0.0039 lb/cm³
k/ρ = 354.0 w-cm²/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 420°K for a Yield Stress of 12,000 psi

Emissivity, ϵ	Number of Fins, N	Fin-Length Parameter, ϕ, cm²-°K⁴/w	Fin-Thickness Parameter, ψ, (cm³-°K⁵/w²)$^{1/2}$	Fin-Weight Parameter, μ, (cm²-lb-°K⁹/w³)$^{1/2}$
1.00	2	8.2855	2.7400	0.049285
	3	6.6536	1.9867	0.039220
	4	5.9651	1.7527	0.037831
	5	5.7623	1.6029	0.038018
	6	5.6200	1.5209	0.039024
	8	5.5936	1.3996	0.041371
0.90	2	9.2663	2.8784	0.054754
	3	7.0492	2.1048	0.042769
	4	6.4434	1.8095	0.040592
	5	6.1526	1.6431	0.040270
	6	5.9094	1.5514	0.040820
	8	5.8263	1.4256	0.043006
0.75	2	11.0350	3.1604	0.065606
	3	8.8649	2.2014	0.050165
	4	7.2735	1.9341	0.046096
	5	6.7303	1.7451	0.044731
	6	6.4795	1.6213	0.044669
	7	6.2896	1.5334	0.044957
	8	6.2419	1.4694	0.045882
0.50	2	17.4190	3.9942	0.104170
	3	12.0580	2.6965	0.071661
	4	10.2320	2.2215	0.062801
	6	8.5048	1.7811	0.056221
	7	8.0949	1.6393	0.054526
	8	7.9412	1.5769	0.055539
	9	7.7944	1.5119	0.055953
	10	7.9902	1.5222	0.060123

Table A4

Optimized Fin Parameters for Magnesium HM21A-T8.

Thermal Conductivity, k = 1.37 w/(cm) (°K)
Density, ρ = 0.0039 lb/cm^3
k/ρ = 351.0 w–cm^2/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 535°K for a Yield Stress of 13,000 psi

Emissivity, ϵ	Number of Fins, N	Fin-Length Parameter, ϕ, cm^2–°K^4/w	Fin-Thickness Parameter, ψ, (cm^3–°K^5/w^2)$^{1/2}$	Fin-Weight Parameter, μ, (cm^2–lb–°K^9/w^3)$^{1/2}$
1.00	2	8.2855	2.7542	0.049540
	3	6.6536	1.9969	0.039423
	4	5.9651	1.7618	0.038027
	5	5.7623	1.6112	0.038214
	6	5.6200	1.5287	0.039226
	8	5.5936	1.4069	0.041585
0.90	2	9.2663	2.8933	0.055038
	3	7.0492	2.1157	0.042990
	4	6.4434	1.8189	0.040802
	5	6.1526	1.6516	0.040478
	6	5.9094	1.5594	0.041031
	8	5.8263	1.4330	0.043228
0.75	2	11.035	3.1768	0.065946
	3	8.8649	2.2128	0.050425
	4	7.2735	1.9441	0.046335
	5	6.7303	1.7541	0.044962
	6	6.4795	1.6297	0.044901
	7	6.2896	1.5413	0.045190
	8	6.2419	1.4770	0.046119
0.50	2	17.419	4.0149	0.104710
	3	12.058	2.7104	0.072032
	4	10.232	2.2330	0.063126
	6	8.5048	1.7904	0.056512
	7	8.0949	1.6478	0.054808
	8	7.9412	1.5851	0.055826
	9	7.7944	1.5197	0.056242
	10	7.9902	1.5300	0.060434

Table A5

Optimized Fin Parameters for Aluminum A356–T6.

Thermal Conductivity, k = 1.59 w/(cm) (°K)
Density, ρ = 0.0059 lb/cm³
k/ρ = 270.0 w-cm²/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 535°K for a Yield Stress of 10,000 psi

Emissivity, ϵ	Number of Fins, N	Fin–Length Parameter, ϕ, cm²-°K⁴/w	Fin–Thickness Parameter, ψ, (cm³-°K⁵/w²)¹/²	Fin–Weight Parameter, μ, (cm²-lb-°K⁹-w³)¹/²
1.00	2	8.2855	2.5565	0.056614
	3	6.6536	1.8536	0.045053
	4	5.9651	1.6353	0.043457
	5	5.7623	1.4955	0.043671
	6	5.6200	1.4190	0.044827
	8	5.5936	1.3059	0.047523
0.90	2	9.2663	2.6857	0.062897
	3	7.0492	1.9638	0.049129
	4	6.4434	1.6883	0.046628
	5	6.1526	1.5331	0.046258
	6	5.9094	1.4475	0.046890
	8	5.8263	1.3301	0.049401
0.75	2	11.0350	2.9487	0.075362
	3	8.8649	2.0540	0.057625
	4	7.2735	1.8045	0.052951
	5	6.7303	1.6282	0.051383
	6	6.4795	1.5127	0.051312
	7	6.2896	1.4307	0.051643
	8	6.2419	1.3710	0.052705
0.50	2	17.4190	3.7267	0.119660
	3	12.0580	2.5159	0.082317
	4	10.2320	2.0728	0.072140
	6	8.5048	1.6619	0.064582
	7	8.0949	1.5295	0.062634
	8	7.9412	1.4713	0.063798
	9	7.7944	1.4106	0.064273
	10	7.9902	1.4202	0.069063

Table A6

Optimized Fin Parameters for Magnesium HM31-T5.

Thermal Conductivity, k = 1.04 w/(cm) (°K)
Density, ρ = 0.004 lb/cm³
k/ρ = 260.0 w-cm²/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 535°K for a Yield Stress of 19,000 psi

Emissivity, ϵ	Number of Fins, N	Fin-Length Parameter, ϕ, cm²-°K⁴/w	Fin-Thickness Parameter, ψ, (cm³-°K⁵/w²)^{1/2}	Fin-Weight Parameter, μ, (cm²-lb-°K⁹/w³)^{1/2}
1.00	2	8.2855	3.1667	0.057411
	3	6.6536	2.2960	0.045686
	4	5.9651	2.0257	0.044068
	5	5.7623	1.8525	0.044286
	6	5.6200	1.7577	0.045457
	8	5.5936	1.6176	0.048192
0.90	2	9.2663	3.3267	0.063782
	3	7.0492	2.4325	0.049820
	4	6.4434	2.0913	0.047284
	5	6.1526	1.8990	0.046909
	6	5.9094	1.7930	0.047549
	8	5.8263	1.6476	0.050096
0.75	2	11.0350	3.6526	0.076422
	3	8.8649	2.5443	0.058436
	4	7.2735	2.2353	0.053696
	5	6.7303	2.0168	0.052106
	6	6.4795	1.8738	0.052034
	7	6.2896	1.7721	0.052369
	8	6.2419	1.6982	0.053446
0.50	2	17.4190	4.6162	0.121350
	3	12.0580	3.1164	0.083476
	4	10.2320	2.5675	0.073155
	6	8.5048	2.0585	0.065491
	7	8.0949	1.8946	0.063515
	8	7.9412	1.8225	0.064695
	9	7.7944	1.7473	0.065177
	10	7.9902	1.7592	0.070035

Table A7

Optimized Fin Parameters for Electrolytic Tough Pitch Copper.

Thermal Conductivity, k = 3.91 w/(cm) (°K)
Density, ρ = 0.019 lb/cm³
k/ρ = 206.0 w-cm²/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 535°K for a Yield Stress of 30,000 psi

Emissivity, ϵ	Number of Fins, N	Fin-Length Parameter, ϕ, cm²-°K⁴/w	Fin-Thickness Parameter, ψ, (cm³-°K⁵/w²)$^{1/2}$	Fin-Weight Parameter, μ, (cm²-lb-°K⁹/w³)$^{1/2}$
1.00	2	8.2855	1.62950	0.065751
	3	6.6536	1.18150	0.052323
	4	5.9651	1.04240	0.050470
	5	5.7623	0.95328	0.050719
	6	5.6200	0.90449	0.052061
	8	5.5936	0.83239	0.055193
0.90	2	9.2663	1.71190	0.073048
	3	7.0492	1.25180	0.057058
	4	6.4434	1.07610	0.054153
	5	6.1526	0.97720	0.053724
	6	5.9094	0.92266	0.054457
	8	5.8263	0.84782	0.057374
0.75	2	11.0350	1.87960	0.087525
	3	8.8649	1.30930	0.066925
	4	7.2735	1.15020	0.061497
	5	6.7303	1.03780	0.059675
	6	6.4795	0.96423	0.059593
	7	6.2896	0.91192	0.059977
	8	6.2419	0.87389	0.061211
0.50	2	17.4190	2.37540	0.138980
	3	12.0580	1.60370	0.095602
	4	10.2320	1.32120	0.083782
	6	8.5048	1.05930	0.075005
	7	8.0949	0.97492	0.072742
	8	7.9412	0.93784	0.074094
	9	7.7944	0.89914	0.074646
	10	7.9902	0.90527	0.080209

Table A8

Optimized Fin Parameters for Chromium–Copper.

Thermal Conductivity, k = 3.23 w/(cm) (°K)
Density, ρ = 0.02 lb/cm^3
k/ρ = 162.0 w-cm^2/(lb) (°K)
τ = 0.99
θ = 0
Maximum Usable Temperature = 866°K for a Hield Stress of 17,000 psi

Emissivity, ϵ	Number of Fins, N	Fin–Length Parameter, ϕ, cm^2–°K^4/w	Fin–Thickness Parameter, ψ, (cm^3–°K^5/w^2)$^{1/2}$	Fin–Weight Parameter, μ, (cm^2–lb–°K^9/w^3)$^{1/2}$
1.00	2	8.2855	1.79180	0.072133
	3	6.6536	1.29920	0.057402
	4	5.9651	1.14620	0.055369
	5	5.7623	1.04820	0.055642
	6	5.6200	0.99456	0.057115
	8	5.5936	0.91528	0.060550
0.90	2	9.2663	1.88240	0.080138
	3	7.0492	1.37640	0.062596
	4	6.4434	1.18330	0.059410
	5	6.1526	1.07450	0.058939
	6	5.9094	1.01450	0.059743
	8	5.8263	0.93226	0.062943
0.75	2	11.0350	2.06670	0.096020
	3	8.8649	1.43960	0.073421
	4	7.2735	1.26480	0.067466
	5	6.7303	1.14120	0.065468
	6	6.4795	1.06030	0.065378
	7	6.2896	1.00270	0.065799
	8	6.2419	0.96092	0.067152
0.50	2	17.4190	2.61200	0.152470
	3	12.0580	1.76340	0.104880
	4	10.2320	1.45280	0.091914
	6	8.5048	1.16480	0.082285
	7	8.0949	1.07200	0.079803
	8	7.9412	1.03120	0.081286
	9	7.7944	0.98868	0.081892
	10	7.9902	0.99542	0.087995

Table A9

Optimized Fin Parameters for Molybdenum.

Thermal Conductivity, k = 1.84 w/(cm) (°K)

Density, ρ = 0.023 lb/cm^3

k/ρ = 80 w-cm^2/(lb) (°K)

τ = 0.99

θ = 0

Maximum Usable Temperature = 1140°K for a Yield Stress of 30,000 psi

Emissivity, ϵ	Number of Fins, N	Fin–Length Parameter, ϕ, cm^2-°K^4/w	Fin–Thickness Parameter, ψ, (cm^3-°K^5/w^2)$^{1/2}$	Fin–Weight Parameter, μ, (cm^2-lb-°K^9/w^3)$^{1/2}$
1.00	2	8.2855	2.3729	0.102640
	3	6.6536	1.7205	0.081677
	4	5.9651	1.5179	0.078784
	5	5.7623	1.3882	0.079173
	6	5.6200	1.3171	0.081268
	8	5.5936	1.2121	0.086156
0.90	2	9.2663	2.4928	0.114030
	3	7.0492	1.8228	0.089068
	4	6.4434	1.5671	0.084534
	5	6.1526	1.4230	0.083863
	6	5.9094	1.3436	0.085008
	8	5.8263	1.2346	0.089561
0.75	2	11.0350	2.7370	0.136630
	3	8.8649	1.9065	0.104470
	4	7.2735	1.6750	0.095997
	5	6.7303	1.5113	0.093153
	6	6.4795	1.4041	0.093025
	7	6.2896	1.3279	0.093624
	8	6.2419	1.2726	0.095550
0.50	2	17.4190	3.4591	0.216940
	3	12.0580	2.3353	0.149240
	4	10.2320	1.9239	0.130780
	6	8.5048	1.5425	0.117080
	7	8.0949	1.4197	0.113550
	8	7.9412	1.3657	0.115660
	9	7.7944	1.3093	0.116520
	10	7.9902	1.3183	0.125210

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